The Macroeconomics of *TANSTAAFL*[*]

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June 19, 2013

Abstract

Market imperfections may lead to underinvestment in dynamic general equilibrium models. An interesting but unexplored question is whether policy interventions which attenuate underinvestment gaps necessarily imply that consumption will initially decline. By employing a calibrated version of a standard R&D-based growth model, we show that raising the R&D subsidy rate may not only close the R&D underinvestment gap but also raise consumption per capita at all times ("intertemporal free lunch"). We also discuss the general mechanics of such an intertemporal free lunch in both one-sector and multi-sector growth models and further examples.

Key words: Intertemporal free lunch; Endogenous growth; R&D underinvestment; Transitional dynamics.

JEL classification: O41, E20, H20.

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[*] *Acknowledgements:* We are grateful to Dirk Krueger and Paul Romer for helpful comments and suggestions. We also benefitted from comments and suggestions by participants at the annual conference of Verein für Socialpolitik in Göttingen, the conference on "Dynamics, Economic Growth, and International Trade" (DEGIT XVII) in Milan, the innovation-workshop in Ottobeuren ("41th Ottobeuren-Seminar"), particularly our discussant Jürgen Meckl, and the CESifo Area conference in Public Sector Economics in Munich, particularly our discussant Ngo Van Long.

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1 Introduction

The saying *There ain't no such thing as a free lunch*, popularized in economics by Milton Friedman (1975), expresses the insight that every benefit comes at a cost. There is one general exception to this rule. If resources are being used inefficiently, it is possible to get "something for nothing". There are some examples of static free lunches, like the gains from trade when opening up goods market from autarky or efficiency gains after addressing static externalities by policy interventions.

To assess the welfare effects of policy interventions in dynamic models, one must take the entire transition path into account. Comparing steady states only may lead to drastically misleading results, especially if the speed of convergence is low. In addition, it would be interesting to know whether a policy intervention could implement a reallocation of resources in a decentralized economy such that per capita consumption increases for some periods and does not decline for any period. Such a possibility may be referred to as *intertemporal free lunch*.

In a dynamic context, it is natural to focus on an intertemporal free lunch associated with investment distortions. The macroeconomics literature on this issue has dealt with the phenomenon of dynamic inefficiency (e.g. Phelps, 1966; Cass, 1972; Abel et al., 1989). Dynamic inefficiency is typically said to occur when a production factor is *over*accumulated in the sense that less investment into that factor than in market equilibrium would lead to an intertemporal free lunch. For instance, as is well-known, the Solow model exhibits dynamic inefficiency if the saving rate lies above its golden rule level such that capital is overaccumulated. In an overlapping-generations context, dynamic inefficiency may result since current generations do not take changes in the future interest rate into account when deciding on their saving rate (Weil, 2008).

By contrast, this paper examines the question whether an intertemporal free lunch is possible if a production factor is *under*accumulated in long run market equilibrium. We explore if a policy intervention in a decentralized economy targeted to increase investment of the underaccumulated factor can lead to a reallocation of resources such that an intertemporal free lunch is realized. To the best of our knowledge, the previous literature has not dealt with this possibility. A priori, one may think that, in a closed economy, higher investment would always imply consumption losses in the short run, even under investment inefficiencies. In fact, if there is a single investment good, higher investment necessarily means lower consumption in the short run (i.e. for given income) in macroeconomic equilibrium.

We employ calibrated versions of the model by Romer (1990), where growth is fully endogenous, and the semi-endogenous growth model by Jones (1995) to show that an intertemporal free lunch may be possible due to multiple investment possibilities. As first demonstrated by Jones and Williams (2000) in a calibrated version of the semi-endogenous growth model, in the long run, a social planner would like to allocate more resources towards R&D than is the case in decentralized equilibrium. The reason is that positive R&D externalities like an intertemporal knowledge spillover outweigh possible negative R&D externalities under empirically motivated calibrations. We show that, by raising the current R&D subsidy rate in a situation with underinvestment, households immediately decrease their saving rate due to their expectation of future productivity advances when labor is reallocated towards R&D in response to the policy intervention. Thus, the accumulation of physical capital slows down in the initial transition phase to the new steady state while more resources are devoted to knowledge accumulation. The decrease in the rate of investment in physical capital then enables an intertemporal free lunch despite a drop in initial per capita income. In our calibrated economy, only for large increases in the R&D subsidy rate, possibly beyond the socially optimal *long run* rate, per capita consumption drops initially.

As we calibrate our economy to the US and deliberately analyze a widely-accepted, standard growth set ups, our results suggest that an intertemporal free lunch is more than a theoretical anomaly and may be realized in advanced economies. We start out with the semi-endogenous growth model of Jones (1995), in which the long run growth rate is independent of the R&D subsidy. That an intertemporal free lunch even occurs in such a framework may thus be considered as a strong argument to raise the R&D subsidy from its current level. In view of some criticisms of semi-endogenous growth theory (e.g. Laincz and Peretto, 2006, p. 268f.), we also examine the seminal framework of Romer (1990) to support and compare our main result in an alternative framework.

From a more general point of view, we show that a necessary condition for an intertemporal free lunch to occur in a situation with underinvestment is that at least two allocation variables can be affected independently by a policy intervention. In the Romer-Jones model, an increase in the R&D subsidy rate induces a reallocation of labor towards the R&D sector. This requires a first allocation variable to be set freely. For an intertemporal free lunch to be feasible, i.e. for consumption not to decrease initially, capital accumulation has to decelerate, which requires a second degree of freedom. This response to the R&D policy intervention allows for consumption smoothing in the presence of a substantial positive wealth effect. We discuss analogous considerations for one-sector and multi-sector growth models.

Technically, to identify an intertemporal free lunch and its underlying causes in sophisticated endogenous growth models requires to numerically compute the entire transition path to the new steady state after a policy shock in non-linear, highly dimensional, saddle-point stable, differential-algebraic systems. Simulating such a dynamic model is all but trivial. The growth literature has used the techniques of linearization, time elimination, or backward integration. Linearization delivers bad approximations if the deviation from the steady state is large, time elimination does not work if there are non-monotonic adjustments, and backward integration fails in case of stiff differential equations. Typically, all of these problems are present in our context. We employ a recent procedure, called relaxation algorithm (Trimborn, Koch and Steger, 2008), which can deal with these conceptual difficulties.

In line with seminal papers in growth theory, we focus on the standard assumption of an infinite planning horizon. As formalized by Barro (1974), the assumption may capture intertemporal utility of short-lived individuals with dynastic bequest motives. For evaluating the dynamic impact of policy interventions, it is thus important to know whether an intertemporal free lunch is conceivable or if instantaneous utility necessarily falls for some periods in response to the policy intervention. In fact, for a long time, scholars felt uneasy with the idea that present generations should give up consumption for the benefit of future generations, including Rawls (1971). For instance, this utilitarian idea is questioned in the debate on natural resource depletion and climate change, which is often based on infinite horizon models as well. Recently, Long and Martinet (2012) proposed a new approach to intertemporal natural resource allocation problems which serves as a different alternative to a utilitarian treatment of different generations: in addition to standard intertemporal welfare, a social planner should ensure that certain minimum thresholds for consumption and resource stocks are met, which is accomplished by introducing an "index of rights" in the objective function (Martinet, 2011). More generally, applied to policy interventions in a dynamic context like ours, this could mean that addressing underinvestment should be (and in a democratic society may be) supported only if the instantaneous change in per capita consumption is nonnegative. The knowledge of whether or not an intertemporal free lunch is feasible therefore could be a crucial information for policy makers.

The focus of our paper is hence very different from the literature on optimal growth policy which maximizes steady state welfare or seeks to identify the policy reform which maximizes the gain in intertemporal welfare which results from it. For instance, Grossmann, Steger and Trimborn (2010) propose a comprehensive semi-endogenous growth model to derive the optimal tax deductions for capital costs, R&D expenditure and human capital expenditure. Grossmann, Steger and Trimborn (2013) show that implementing the optimal long run R&D subsidy rate has large welfare gains which are almost as high than implementing the dynamically optimal path for R&D subsidizations. Grossmann and Steger (2013) show that allowing for heterogeneity of R&D skills leaves the analytical solution for the optimal long run subsidy mix unaffected. None of these papers examine the possibility of an intertemporal free lunch, however, strictly focussing on the utilitarian paradigm instead.

The paper is organized as follows. Section 2 shows that an intertemporal free lunch exists in the Romer-Jones model when we calibrate it to the US. Section 3 derives necessary conditions of an intertemporal free lunch in dynamic, closed economy, representative agent frameworks. In section 4, we discuss the possibility of an intertemporal free lunch in specific endogenous growth models other than the Romer-Jones model. The last section concludes.

2 Intertemporal free lunch in the Romer-Jones model

This section illustrates the possibility of an intertemporal free lunch in a widely-used R&D-based growth model with both accumulation of knowledge and physical capital goods. We start with a definition of an intertemporal free lunch in the context of representative agent models and a possible role for policy intervention. Time is indexed by $t \in \mathbb{R}$.

Definition 1. *Let* $c_A(t)$ *denote the time path of consumption of a representative agent under the status quo policy and let* $c_B(t)$ *denote the time path of per capita consumption after a policy change. An intertemporal free lunch is possible if and only if there is a feasible policy measure such that* $c_B(t) > c_A(t)$ *for at least some* t *and* $c_B(t) \geq c_A(t)$ *for all* t *in a decentralized economy.*

Notice that we require that an intertemporal free lunch can be realized by a *feasible policy intervention* in a market equilibrium.

2.1 The Romer-Jones model

Consider an R&D-based growth model which heavily draws on Romer (1990) and Jones (1995). There is mass one of infinitely-lived households of size N. Each household supplies one unit of time to a perfect labor market (i.e., total employment is equal to N). Initially, $N(0) = N_0 > 0$. Household size grows at constant exponential rate $n \geq 0$. We employ the standard intertemporal utility function of the representative household

$$U = \int_0^\infty u(c(t))e^{-(\rho-n)t}\mathrm{d}t, \tag{1}$$

$\rho > 0$, where $c(t)$ is per capita consumption at time t and u denotes instantaneous utility, given by $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$, $\sigma > 0$. Let w and r denote the wage rate and the interest rate, respectively. The economy is closed such that factor prices are endogenous. Financial wealth per individual, a, accumulates according to[1]

$$\dot{a} = (r - n)a + w - c - T, \tag{2}$$

where $a_0 > 0$ is given and T is a possible lump-sum tax which finances investment subsidies (introduced below).[2] We assume that the government budget is balanced each period. This restriction ensures that, when underinvestment problems are tackled, an intertemporal free lunch does not arise from incurring debt in early transition phases.

Final output Y is produced according to

$$Y = (L^Y)^{1-\alpha} \int_0^{A^Y} (x_i)^\alpha \mathrm{d}i, \tag{3}$$

$0 < \alpha < 1$, where L^Y denotes labor employed in final output production and x_i the quantity of (physical) capital good $i \in [0, A^Y]$ demanded by the representative final goods producer. One unit of final output can be transformed into one unit of each capital good and all capital goods depreciate at the same constant rate $\delta \geq 0$.

The number of intermediate goods supplied in this economy is denoted by A ("stock of knowledge"). Like physical capital, it is an accumulable factor which expands through horizontal innovations according to

$$\dot{A} = \tilde{\nu} A^\phi L^I \tag{4}$$

[1] $\dot{Z}$ denotes the derivative of a variable Z with respect to time. The time index is omitted whenever this does not lead to confusion.

[2] The assumption of lump sum taxation to finance R&D subsidies is common in the endogenous growth literature. In the present context, it ensures that the first best allocation can be implemented. Grossmann et al. (2013) show that the same is true if taxes on income from financial assets and labor are used instead of lump sum taxation. In any case, the mechanics of an intertemporal free lunch does not depend on the form of taxation. By assuming lump sum taxes we keep the analysis as simple as possible.

with $\phi \leq 1$, $0 \leq \theta < 1$, $\tilde{\nu} := \nu \left(L^I\right)^{-\theta}$, $\nu > 0$, where L^I is labor employed in innovative activities ("R&D") and $\tilde{\nu}$ is taken as given by the representative R&D firm. The stock of knowledge A enters as *non-rival* input into the knowledge accumulation process. Thus, parameter ϕ measures the extent of an intertemporal knowledge spillover (which is positive if $\phi > 0$) and labor is the only rival R&D input. An increase in θ means a larger wedge between the privately perceived constant-returns to R&D and the socially declining marginal product of R&D labor investment.

We examine and compare two cases. First, the case where $\phi < 1$ and possibly $n > 0$, $\theta > 0$, as in Jones (1995). Second, the case where $\phi = 1$ and $\theta = n = 0$, as in Romer (1990).

Both the market for the final good and the labor market are perfect. Also the R&D sector is perfectly competitive. Physical capital good producers possess market power but are restricted by a competitive fringe of firms which do not allow them to charge a mark-up higher than $\kappa \in (1, 1/\alpha]$.[3] Initially, $x_i(0) = x_0 > 0$ for all i and $A(0) = A_0 > 0$.

The government may subsidize (or tax) costs of both R&D firms and capital good producers at time-invariant rates s_A and s_K, respectively.[4] Subsidies are financed by a lump-sum tax levied on households.

2.2 Equilibrium

We can define total manufacturing capital demand as $K^Y := \int_0^{A^Y} x_i \mathrm{d}i$. Capital supply, K, evolves according to final goods market clearing condition $\dot{K} = Y - Nc - \delta K$. In equilibrium, $A^Y = A$ and $K^Y = K$ with $K_0 = A_0 x_0 > 0$. Moreover, under symmetry, $x_i = x = K^Y/A^Y = K/A$ for all i. Defining $y := Y/N$, $l^Y := L^Y/N$, $k := K/N$,

[3] See e.g. Aghion and Howitt (2005). In addition to introducing investment subsidies, this is the only way we depart from Jones (1995) in this section. We introduce the competitive fringe in order to calibrate the mark-up factor according to empirical estimates.

[4] Grossmann et al. (2013) derive the optimal time paths of subsidies on R&D and capital costs in the proposed model and find that optimal subsidy rates should change little over time.

according to (3), per capita output reads

$$y = k^{\alpha}(Al^{Y})^{1-\alpha}. \tag{5}$$

Moreover, the capital-labor ratio, k, evolves according to

$$\dot{k} = k^{\alpha}(Al^{Y})^{1-\alpha} - (n+\delta)k - c, \tag{6}$$

Define by $inv := (\dot{K} + \delta K)/Y$ the economy's aggregate investment rate in physical capital and by $q := c/y$ the consumption rate. For later use, (5) can also be written in shares which capture the use of final output:

$$q + inv = 1. \tag{7}$$

Denote by p_i the price of capital good i and by P^A the price of blueprints for new physical capital goods. The profit of the representative R&D firm is $\Pi = P^A \tilde{\nu} A^{\phi} L^I - (1 - s_A)\, wL^I$. Profits of capital producer i are $\pi_i = [p_i - (1 - s_K)(r + \delta)]\, x_i$. Accounting for the competitive fringe, any capital producer i sets the price to

$$p_i = \kappa(1 - s_K)(r + \delta). \tag{8}$$

Thus, all intermediate goods producers have the same profit due to the symmetry in their sector, i.e. $\pi_i = \pi$ for all i. As will become apparent, mark-up pricing distorts capital accumulation.

The equilibrium is defined as follows.

Definition 2. *A market equilibrium consists of time paths for the quantities* $\{L_t^I, L_t^Y, c_t, \{x_{it}\}_{i=0}^{A}, a_t, Y_t, K_t, A_t\}_{t=0}^{\infty}$ *and prices* $\{P_t^A, \{p_{it}\}_{i=0}^{A}, w_t, r_t\}_{t=0}^{\infty}$ *such that final goods producers, intermediate goods producers and R&D firms maximize profits; households maximize intertemporal welfare; the capital market equilibrium condition,* $\dot{P}_t^A/P_t^A + \pi/P_t^A = r_t$, *holds; the labor market clears,* $L_t^Y + L_t^I = N_t$; *the financial* *market clears,* $a_t N_t = K_t + P_t^A A_t$*, where* $K_t = K_t^Y = \int_0^A x_{it} di$*; goods markets clear; the government budget is balanced.*

Define $p^A := P^A/N$ and $l^I := L^I/N$. In the case $\phi < 1$ (Jones, 1995), we have $\dot{l}^I = \dot{p}^A = 0$ in steady state. Thus, as is well-known,

$$\frac{\dot{k}}{k} = \frac{\dot{c}}{c} = \frac{\dot{A}}{A} = \frac{\dot{y}}{y} = \frac{(1-\theta)n}{1-\phi} \equiv g \tag{9}$$

holds in the long run. That is, the long run growth rate of per capita income is independent of the R&D subsidy rate, s_A, and independent of population size ("scale"), N, while increasing in the population growth rate, n.

By contrast, when $\phi = 1$ and $\theta = n = 0$ (Romer, 1990), economic growth is stimulated by increasing s_A even in the long run, as is also well-known. Moreover, economic growth is fostered by an increase in N. This scale effect property has been widely criticized (e.g. Jones, 2005). As shown in the online-appendix, the long run growth rate reads as follows:

$$\frac{\dot{k}}{k} = \frac{\dot{c}}{c} = \frac{\dot{A}}{A} = \frac{\dot{y}}{y} = \nu N \frac{(1-1/\kappa)\,\nu N - (1-s_A)\,(1/\alpha - 1)\,\rho}{1 - 1/\kappa + (1-s_A)\,(1/\alpha - 1)\,\sigma \nu N} \tag{10}$$

2.3 Optimal Long Run Growth Policy

In order to identify potential underinvestment in R&D and physical capital, we have to compare the decentralized equilibrium with the social planning optimum. From this, we can derive which subsidy rates s_A and s_K implement the first-best optimum.

From (4), the socially relevant evolution of the knowledge stock is

$$\dot{A} = \nu A^{\phi} (N l^I)^{1-\theta}. \tag{11}$$

Expressed in employment shares, the labor resource constraint implies

$$l^Y + l^I = 1. \tag{12}$$

The social planner maximizes intertemporal welfare U subject to (6), (11) and (12), and non-negativity constraints, where c, l^Y, l^I are control variables and k, A are state variables. It is easy to show that in the social planning optimum the same steady state properties as in decentralized equilibrium hold.

Proposition 1.

(a) If $\phi < 1$ ("Jones-model"), then one can implement the first-best allocation of labor and the first-best investment rate in the long run by setting subsidy rates to

$$s_K = 1 - \frac{1}{\kappa} \equiv s_K^*, \tag{13}$$

$$s_A = 1 - \frac{1 - 1/\kappa}{1/\alpha - 1} \frac{(\sigma - 1)g + \rho - \theta n}{(1 - \theta)(\sigma g + \rho - n)} \equiv s_A^*. \tag{14}$$

(b) If $\phi = 1$ and $n = \theta = 0$ ("Romer-model"), then the optimal long run subsidy rates are given by $s_K = s_K^*$ and

$$s_A = 1 - \frac{1 - 1/\kappa}{1/\alpha - 1} \left[1 - \frac{1}{\sigma} \left(1 - \frac{\rho}{\nu N} \right) \right] \equiv s_A^{**}. \tag{15}$$

Proof: See online-appendix.

To get an idea why there may be an intertemporal free lunch for the case $\phi < 1$ (Jones, 1995),[5] suppose the economy is in steady state and there is underinvestment in R&D, but not in physical capital; that is, compared to the social planner's solution, the long run fraction of labor devoted to R&D, l^I, is too low in market equilibrium, such that $s_A < s_A^*$ and the long run investment rate is socially optimal, $s_K = s_K^*$. The latter assumption ensures that the possibility of an intertemporal free lunch is not driven by overaccumulation of physical capital, unlike in the literature on dynamic inefficiency.

As is well-known, even in the case where there are no externalities in the knowledge accumulation process (4), i.e. $\phi = \theta = 0$, accumulation of A is distorted downwards since innovators cannot fully appropriate the social gain from an innovation.[6] To

[5] For the Romer-model ($\phi = 1$, $n = \theta = 0$) the reasoning is similar.

[6] That is, in equilibrium, the profit of an intermediate good firm, π, is smaller than the social return to an additional intermediate good, $\partial Y / \partial A$.

see why it may be possible to realize an intertemporal free lunch by increasing R&D subsidy rate, s_A, consider the following. On the one hand, there will be an immediate reallocation of labor from manufacturing (decrease in $l^Y(0)$) to R&D (increase in $l^I(0)$). This lowers initial per capita income, $y(0) = Y(0)/N_0$. On the other hand, however, the aggregate investment rate, inv, may decrease initially. According to (7), this means that the initial consumption rate $q(0)$ increases. If this latter effect is strong enough, the initial consumption level, $c(0) = q(0)y(0)$, increases despite higher R&D investment.

2.4 Calibration

We calibrate the economy to largely match the characteristics of the US economy under the assumption that the US is in steady state.[7] We take a per capita long run output growth rate (g) of 2 percent and a long run interest rate (r) of 7 percent. Given the time preference ρ, parameter σ is determined by the Keynes-Ramsey rule $\dot{c}/c = g = (r - \rho)/\sigma$. The capital depreciation rate δ can be inferred from the US investment rate (inv) and the capital-output ratio (K/Y). We use that $inv = (\dot{K} + \delta K)/Y = (\dot{K}/K + \delta)K/Y$. In steady state, $\dot{K}/K = n + g$; thus, $inv = (n + g + \delta)K/Y$. For the US, we observe $inv = 0.21$ and $K/Y = 3$. Moreover, we assume a population growth rate (n) of 1 percent, leading to $\delta = 0.04$. We keep this value throughout, although $n = 0$ in the Romer-model. Our results are not sensitive to changes in δ.

To focus on R&D underinvestment, we also assume that the capital subsidy rate ensures optimal capital investment at all times, $s_K = s_K^* = 1 - 1/\kappa$. Following Jones and Williams (2000) and Chang, Hung and Huang (2011), the mark up factor κ is set to 1.4, which is at the upper end of the range suggested by the empirical estimates by Norrbin (1993). We confirmed that our results are quite insensitive to the value of κ and become even more pronounced if we lower κ. A higher mark up factor mitigates the well known "surplus appropriability problem", which gives rise to R&D underinvestment, but aggravates underinvestment in physical capital. Setting $\kappa = 1.4$ implies $s_K^* = 2/7$. The output elasticity of capital (α) is given by the condition that the user cost of

[7] The calibration strategy for the case $\phi < 1$ largely follows Grossmann et al. (2010, 2013).

capital $(r+\delta)$ equals the marginal product of capital under optimal capital investment: $\alpha = (r+\delta)K/Y = 0.33$. In the Jones-model, the R&D underinvestment problem, if present, is enhanced the higher is ϕ and the lower is θ. If θ is close to one, there may be R&D overinvestment. We assume an intermediate value $\theta = 0.5$. Matching the economy's steady state growth rate g in (9), implies that $\phi = 1-(1-\theta)n/g = 0.75$. The current US R&D subsidy rate is just slightly above zero ($s_A = 0.066$). Table 1 below summarizes our calibration.

In the Jones-model, the R&D intensity which is implied by the calibration is as observed for business R&D in the US: $wL^I/Y \simeq 0.02$. Using the parameter values from Table 1 in (14), we find that the optimal R&D subsidy rate which implements the optimal long run labor allocation is fairly high: $s_A^* = 0.79$. It reflects severe R&D underinvestment. The sectoral misallocation of labor is the underlying source for the possibility of an intertemporal free lunch. The optimal rate $s_A = s_A^* = 0.79$ implies a (first best) R&D intensity of about 15 peercent in the long run.[8]

Parameter	Value	Source	Parameter	Value	Source
g	0.02	PWT 6.2 (Heston et al., 2006)	σ	2.5	implied
n	0.01	PWT 6.2 (Heston et al., 2006)	κ	1.4	Chang et al. (2011)
δ	0.04	implied	s_A	0.066	OECD (2009)
r	0.07	Mehra and Prescott (1985)	s_K	2/7	first best value
α	0.33	implied	θ	0.5	intermediate value
ρ	0.02	"usual value"	ϕ	0.75	implied

Table 1: Calibration in the Jones-model

In the Romer-model, we take the same parameter values as in the Jones-model except, of course, using $\phi = 1$ and $n = \theta = 0$. To calibrate the term νN which enters the expression for the optimal R&D subsidy rate, s_A^{**} in (15), we assume again that

[8] Grossmann et al. (2010, 2013) derive similar values for the behaviorally subsidy rates when accounting for (i) endogenous human capital accumulation, (ii) distortionary income taxation, (iii) business stealing effects from R&D (following Jones and Williams, 2000), (iv) transitional dynamics, and (v) a more general production function for final output.

the US is in steady state; that is, $\frac{\dot{y}}{y} = \frac{\dot{A}}{A} = g = 0.02$. As $\frac{\dot{A}}{A} = \nu N l^I$ when $\phi = 1$ and $\theta = 0$, according to (11), in steady state, $l^I = \frac{g}{\nu N}$ holds. In the online-appendix, we show that this fraction of R&D labor is consistent with a long run equilibrium under the current US R&D subsidy rate $s_A = 0.066$ when $\nu N = 0.48$. Using this together with the parameter values given in Table 1 then implies $s_A^{**} = 0.91$, according to (15). Thus, the optimal long run R&D subsidy is higher in the Romer-model than in the Jones-model, reflecting the result that the long run growth rate rises with s_A in the Romer-model but does not affect g in the Jones-model.

2.5 The Intertemporal Free Lunch

We now analyze the equilibrium dynamics for a given (non-optimal) R&D subsidy rate (s_A) and the impact of a change in s_A numerically. The numerical simulations rest on calibration of the model to the US. Transitional dynamics are calculated numerically by applying the relaxation algorithm (Trimborn et al., 2008).[9]

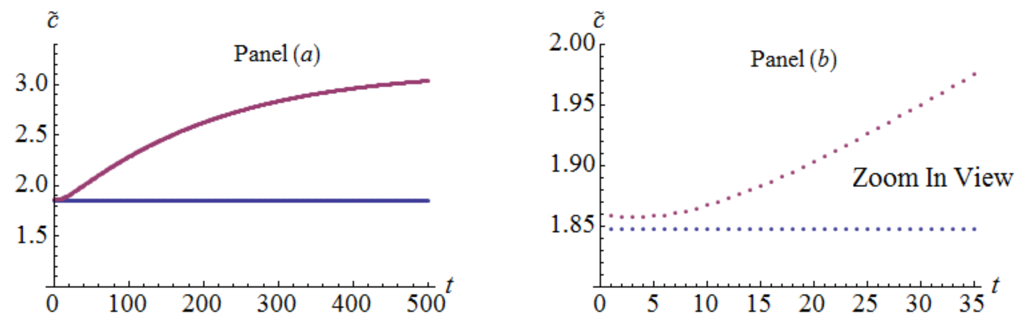

Figure 1: The intertemporal free lunch in the Jones model

It is assumed that the economy is in a steady state initially with $s_A = 0.066$. Fig. 1 shows the time path of the detrended per capita consumption level, $\tilde{c} := c/N^{\frac{1-\theta}{1-\phi}}$, in response to an increase to $s_A = 0.3$. We see that $\tilde{c}$ jumps instantaneously above its pre-shock steady state level (horizontal line) and remains above that value along the entire transition path.

[9]The algorithm is implemented in Mathematica and MatLab. The underlying files are available from the authors upon request.

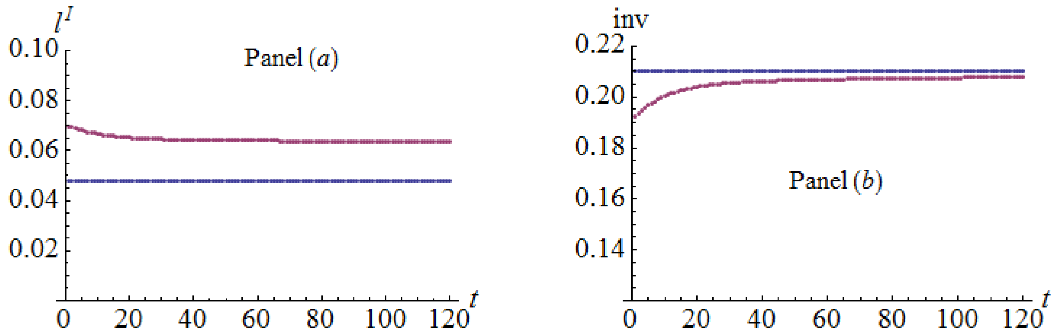

Figure 2: The response of basic allocation variables in the Jones-model

The mechanism which drives this intertemporal free lunch is as follows. The increase in s_A causes the fraction of labor allocated to R&D, $l^I = 1 - l^Y$, to increase. According to panel (a) of Fig. 2, l^I instantaneously jumps from the initial steady state level slightly above the new steady state value, monotonically decreasing along the transition. Due to the decline in the fraction of labor in manufacturing (l^Y), per capita output y drops initially (labor reallocation effect). However, the expansionary R&D policy attenuates the substantial R&D underinvestment in the market economy. Rational, forward-looking agents understand that there is an associated wealth effect. They therefore reduce the fraction of output devoted to the accumulation of physical capital. According to panel (b) of Fig. 2, the aggregate investment rate in physical capital, inv, instantaneously decreases considerably after the policy shock (monotonically increasing towards the old steady state value during the transition). Consequently, the rate of consumption, $q = 1 - inv$, rises initially. For the policy shock we considered, the increase in $q(0)$ is large enough such that, despite the decrease in initial per capita income, $y(0)$, per capita consumption $c(0) = q(0)y(0)$ jumps up initially.

How does the proportional initial change of consumption depend on the policy instrument s_A? To see this, consider the initial rate of change (at $t = 0$) of detrended per capita consumption, $\Delta\tilde{c}(0)/\tilde{c}(0)$, in response to a change in s_A from $s_A = 0.066$ to $s_A \in [0.066 - 0.3, 0.066 + 0.75]$. By construction, at initial (US) value $s_A = 0.066$, we have $\Delta\tilde{c}(0) = 0$. Panel (a) of Fig. 3 shows $\Delta\tilde{c}(0)/\tilde{c}(0)$ as function of the R&D subsidy.

We see that $\Delta\tilde{c}(0)/\tilde{c}(0)$ is rising in s_A up to $s_A \simeq 0.52$ and is negative for s_A−increases slightly beyond $s_A \simeq 0.71$. If s_A jumps to 52 percent, as considered in Fig. 1, initial consumption rises by about 1 percent.

To further clarify the economic intuition of an intertemporal free lunch, let us decompose the rate of change of detrended per capita consumption by using the definition of the consumption rate, $q = c/y = \tilde{c}/\tilde{y}$. Using $\tilde{c} = q\tilde{y}$ we have

$$\frac{\Delta\tilde{c}(0)}{\tilde{c}(0)} = \frac{\Delta q(0)}{q(0)} + \frac{\Delta\tilde{y}(0)}{\tilde{y}(0)} + \frac{\Delta q(0)}{q(0)} \cdot \frac{\Delta\tilde{y}(0)}{\tilde{y}(0)}. \tag{16}$$

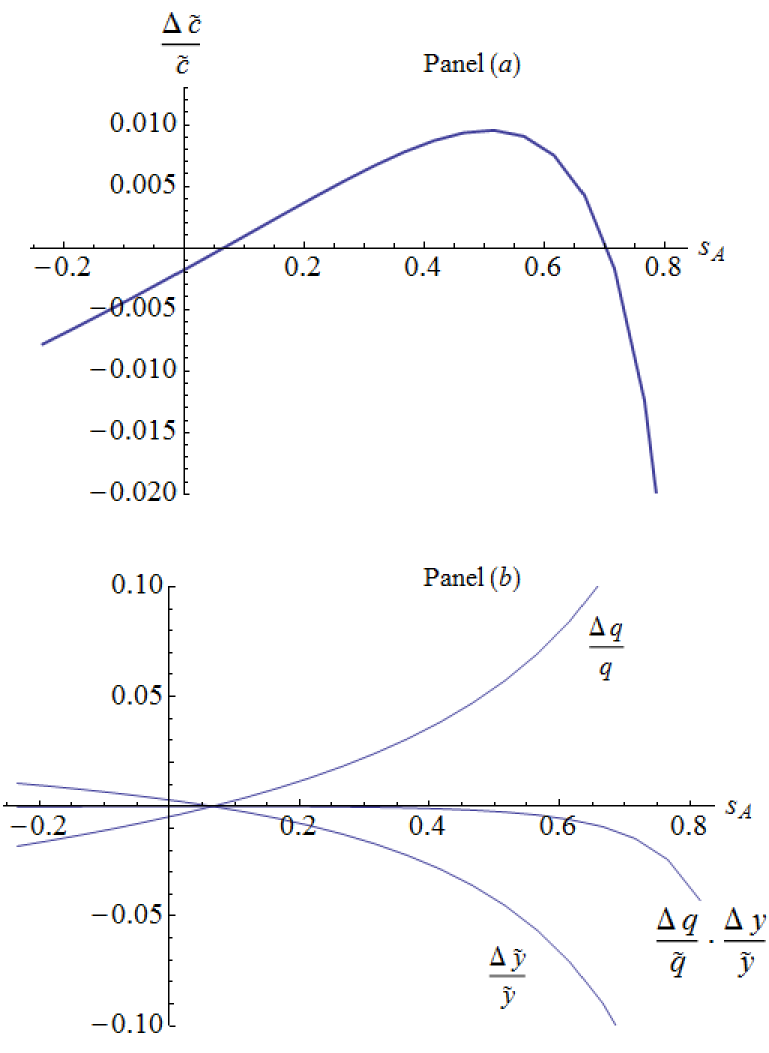

Figure 3: The instantaneous impact of an increase in the R&D subsidy in the Jones-model

Panel (b) of Fig. 3 shows the three terms on the right-hand side of the preceding equation as function of s_A. We confirm that, for $s_A > 0.066$, the proportional change in the consumption rate $q(0)$ is positive and increasing in s_A, whereas the proportional change in (detrended) per capita output $y(0)$ is negative and decreasing in s_A. When s_A is not too high, the rise in the consumption rate is rather large relative to the drop in per capita output, implying $\Delta\tilde{c}(0) > 0$.

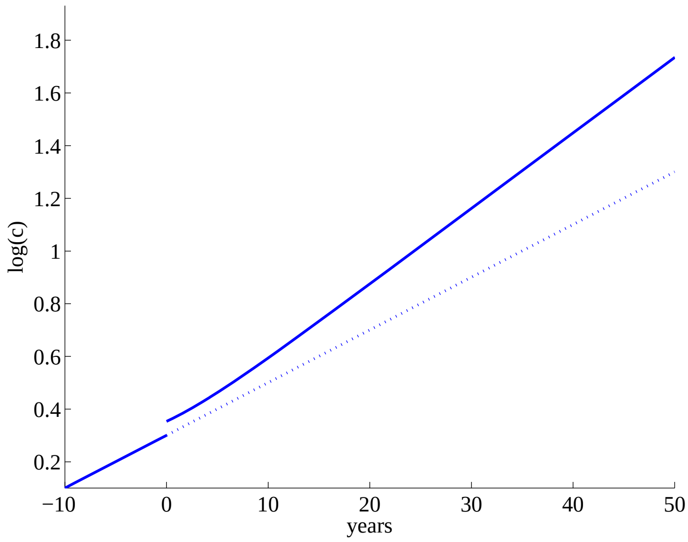

Figure 4: The intertemporal lunch in the Romer model.

Next we illustrates the intertemporal free lunch in the Romer model. As before, it is assumed that the economy starts out from a steady state with $s_A = 0.066$ and that the government increases the R&D subsidy rate permanently to $s_A = 0.3$. Figure 4 shows the time path of (logarithmic) consumption for the R&D subsidy rate held constant at $s_A = 0.066$ (dashed line) and the time path of (logarithmic) consumption assuming that the R&D subsidy rate is increased to $s_A = 0.3$ (solid line).[10] Again, one can clearly observe an intertemporal free lunch. Moreover, the initial increase in consumption $\Delta\tilde{c}(0)/\tilde{c}(0)$ is about 5 percent, which is much higher compared to the

[10]Consumption is not detrended since this policy measure has an impact on the long run growth rate. The calibration is described in section B of the online appendix.

case of the Jones model. Increasing the R&D subsidy rate does now accelerate growth even in the long run such that the wealth effect is strengthened and hence the initial increase in consumption is larger.

3 The Mechanics of an Intertemporal Free Lunch

To generalize the mechanics of an intertemporal free lunch, consider a dynamic, closed economy, representative agent framework with a single consumption good, chosen as numeraire. The production function F of the homogenous final good is

$$Y = F(K_1^Y, K_2^Y, ..., K_I^Y, L^Y), \tag{17}$$

where K_i^Y is the input of capital good $i \in \{1, ..., I\}$ and L^Y is labor input into final production. Capital goods are factors which are accumulable by investments. Labor supply is of size N and non-accumulable (but may grow exogenously).[11]

Total supply of accumulable factor i is denoted by K_i. Given initial level $K_i(0)$, there are $\tilde{I} \leq I$ accumulable factors which evolve according to

$$\dot{K}_i = G_i(Y^i) - \delta_i K_i, \tag{18}$$

where function $G_i(\cdot)$ gives us the gross increase of factor $i \in \{1, ..., \tilde{I}\}$, Y^i denotes the amount of final output devoted to accumulation of factor i and $\delta_i \geq 0$ is the depreciation rate of capital good i. Moreover, there are $I - \tilde{I}$ accumulable factors which evolve according to

$$\dot{K}_i = H_i(L^i) - \delta_i K_i, \tag{19}$$

where L^i is the amount of labor devoted to accumulation of factor $i \in \{\tilde{I} + 1, ..., I\}$. $G_i(\cdot)$ and $H_i(\cdot)$ are increasing functions.[12]

[11] For simplicity, we consider one non-accumulable factor only. Generalization to more than one non-accumulable factor is straightforward.

[12] The analysis becomes more complicated when capital goods are used as inputs for the production of capital goods as well. This and other extensions of the set up are left for future research. The simple set up we focus on here encompasses important workhorses in the study of endogenous growth.

The economy's goods market clearing conditions read as

$$Nc + \sum_{i=1}^{\tilde{I}} Y^i = Y. \tag{20}$$

$$L^Y + \sum_{i=\tilde{I}+1}^{I} L^i = N, \tag{21}$$

$$K_i^Y = K_i \text{ for all } i \in \{1, ..., I\}. \tag{22}$$

Definition 3. *Define the set* $\mathcal{A} = \{c, K_1^Y, ..., K_I^Y, Y^1, ..., Y^{\tilde{I}}, L^{\tilde{I}+1}, ..., L^I, L^Y\}$. *The elements in* $\mathcal{A}$ *are called "allocation variables". Let* V *denote the number of allocation variables and* R *the number of "restrictions" (20)-(22). The difference* $D := V - R$ *is called "degrees of freedom".*

We next examine which role the "degrees of freedom" play for the possibility of an intertemporal free lunch. We start with the following examples.

Example 1. The Romer-Jones model analyzed above is a special case of the more general model proposed in this section. We have $\tilde{I} = 1$ and $I = 2$, i.e., there are two accumulable factors, the first one is produced with forgone consumption (physical capital) and the other one (knowledge) is produced with labor. That is, $K_1 = K$ accumulates according to (18) with $G_1(Y^1) = Y^1 = Y - Nc$: moreover, $\delta_1 = \delta$. The second accumulable factor, $K_2 = K_I = A$, accumulates according to (19) with $H_2(L^I) = \tilde{\nu} A^{\phi} L^I$ and $\delta_2 = 0$. Output reads as $Y = (K^Y)^{\alpha}(A^Y L^Y)^{1-\alpha}$. There are $V = 6$ allocation variables in the set $\mathcal{A} = \{c, K^Y, A^Y, Y^1, L^Y, L^I\}$ and $R = 4$ restrictions: (i) $Nc + Y^1 = Y$ follows from (20). Recalling $q = Nc/Y$ and $inv = (\dot{K} + \delta K)/Y = Y^1/Y$, it gives rise to $q + inv = 1$ as in (7). (ii) $L^Y + L^I = N$ follows from (21) and gives rise to $l^Y + l^I = 1$ as in (12). Finally, we have (iii) $K^Y = K$ and (iv) $A^Y = A$. Thus, there are $D = 2$ degrees of freedom. As we have seen above, the intertemporal free lunch from raising the R&D subsidy comes from the possibility that an increase in allocation variable $l^I = L^I/N$ (which implies a decrease in per capita output) is consistent with an increase in per capita consumption, c, at all times because the investment rate, inv, may decline.

Example 2. Now consider a basic model which captures a "learning-by-doing" externality à la Arrow (1962) and Romer (1986). The production function of a representative final goods producer is

$$Y = a(\bar{K}^Y) \left(K^Y\right)^{\alpha} (L^Y)^{1-\alpha}, \tag{23}$$

$0 < \alpha < 1$, where K^Y denotes aggregate capital input and $a(\bar{K}^Y)$ is an increasing function of the average capital stock, $\bar{K}^Y$, which is taken as given by firms in the $Y-$sector. The assumption captures that final goods producers do not take into account that capital investment raises the economy-wide capital stock and therefore enhances total factor productivity. This externality distorts capital accumulation downwards. The capital stock accumulates like in the Romer-Jones model, i.e., the final goods market clearing condition can be expresed as $q = 1 - inv$. In equilibrium, $K^Y = \bar{K}^Y = K$ and $L^Y = N$ (full employment conditions). For instance, if total labor supply $N = 1$ and $a(\bar{K}^Y) = A \cdot (\bar{K}^Y)^{1-\alpha}$, $A > 0$, the social production function is $Y = AK$ ("AK-model"). Now, addressing the learning-by-doing externality by a policy intervention which raises the investment rate, inv, leaves per capita output initially unaffected but inevitably lowers the initial consumption rate, $q(0)$. Thus, an intertemporal free lunch is *never* possible. Note that there are $V = 4$ allocation variables (c, Y^1, K^Y, L^Y) and $R = 3$ constraints (full employment conditions and final goods market clearing), i.e., $D = V - R = 1$. This shows that with just one degree of freedom, we cannot have an intertemporal free lunch.

If both kinds of constraints (20) and (21) are present ($1 \leq \tilde{I} < I$), as in Example 1, there are $V = 2I + 2$ allocation variables, according to Definition 3, and $R = 2I$ restrictions, according to (20)-(22). Thus, for any number of capital goods, I, there are exactly $D = 2$ degrees of freedom. Consequently, also for $I > 2$, the same logic applies to demonstrate the possibility of an intertemporal free lunch as in the Romer-Jones model (Example 1).

One should note that, generally, the presence of two accumulable factors does not ensure that there are two degrees of freedom. To see this, consider again Example 2, which could be interpreted to encompass two accumulable input factors; first, physical capital (K) and second, total factor productivity $a = a(\bar{K}^Y)$. Using $\bar{K}^Y = K$, we have $\dot{a} = a'(K)\dot{K}$. However, as demonstrated, there is just one degree of freedom. Changes in a and K go in the same direction which is why an intertemporal free lunch is not possible.

The preceding discussion can be summarized by the following proposition.

Proposition 2. *In the class of models considered in this section, a necessary condition for an intertemporal free lunch to occur is that there are at least two degrees of freedom, $D \geq 2$.*

4 Conclusion

Market imperfections may lead to underinvestment in dynamic general equilibrium models. This paper has explored the question whether policy interventions which attenuate underinvestment gaps imply that consumption of households will necessarily decline initially. In this case, Milton Friedman's conjecture *There ain't no such thing as a free lunch* (*TANSTAAFL*) would apply.

By contrast, employing calibrated versions of endogenous growth models with horizontal innovations (Romer, 1990; Jones, 1995), we have shown that raising the R&D subsidy rate may not only close the R&D underinvestment gap but also raise consumption per capita at all times ("intertemporal free lunch"). In particular, we have shown that higher R&D investment, which is induced by an increase in the R&D subsidy rate, goes along with an immediate slowdown in the process of capital accumulation. This market response represents a wealth effect in the consumption-savings decision of households, which can be sufficiently strong to enjoy an intertemporal free lunch.

According to our calibration strategy, our results suggest that an intertemporal free lunch could be realized in advanced economies like the US. Identifying an intertemporal free lunch has the advantage that we do not have to invoke the utilitarian idea that requires current generations to give up consumption for the benefit of future generations in order to achieve an intertemporal welfare gain. Thus, we may conclude that our findings provide a more powerful argument for policy intervention than the previous literature on underinvestment in dynamic macroeconomics. Certainly, however, future research on calibrated versions of alternative classes of dynamic macroeconomic models is required to become more confident that policy interventions which address underinvestment problems in long run market equilibrium are likely to lead to an intertemporal free lunch.

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Online-Appendix

(not intended for publication)

A. The Jones (1995) model

We first derive the dynamic system and then the optimal long run subsidization policy.[13]

Proposition A.1. *Let* $\tilde{z} := z/N^{\frac{1-\theta}{1-\phi}}$ *be the detrended value for any variable* $z \in \{y, k, c, A\}$. *Also suppose that* $\phi < 1$ *and*

$$\rho - n + (\sigma - 1)g > 0 \text{ with } g \equiv \frac{(1-\theta)n}{1-\phi} \tag{A1}$$

hold. Then the evolution of the market economy is governed by the following dynamic system (in addition to appropriate boundary conditions):

$$\begin{aligned}
\dot{\tilde{A}} &= \nu \tilde{A}^{\phi} \left(l^I\right)^{1-\theta} - g\tilde{A}, && (24)\\
\dot{p}^A &= (r-n)\, p^A - \frac{(\kappa-1)\,(\alpha/\kappa)^{\frac{1}{1-\alpha}}\,(1-l^I)}{[(1-s_K)(r+\delta)]^{\frac{\alpha}{1-\alpha}}}, && (25)\\
\dot{\tilde{k}} &= \left(\tilde{A}(1-l^I)\right)^{1-\alpha} \tilde{k}^{\alpha} - \tilde{c} - (\delta+n+g)\,\tilde{k}, && (26)\\
\dot{\tilde{c}} &= \frac{\tilde{c}\,(r-\rho)}{\sigma} - g\tilde{c}, && (27)\\
r+\delta &= \frac{\alpha}{\kappa(1-s_K)} \left(\frac{\tilde{A}(1-l^I)}{\tilde{k}}\right)^{1-\alpha}, && (28)\\
p^A \nu \tilde{A}^{\phi-1} (l^I)^{-\theta} &= (1-s_A)\,(1-\alpha) \left(\frac{\tilde{k}}{\tilde{A}(1-l^I)}\right)^{\alpha}. && (29)
\end{aligned}$$

Proof of Proposition A.1. According to (1) and (2), the current-value Hamiltonian which corresponds to the household optimization problem is given by

$$\mathcal{H} = \frac{c^{1-\sigma}-1}{1-\sigma} + \lambda\left((r-n)a + w - c - T\right), \tag{30}$$

[13] See Grossmann et al. (2012) for a similar analysis.

where λ is the co-state-variable associated with constraint (2). Necessary optimality conditions are $\partial \mathcal{H}/\partial c = 0$, $\dot{\lambda} = (\rho - n)\lambda - \partial \mathcal{H}/\partial a$, and the corresponding transversality condition. Thus,

$$\lambda = c^{-\sigma}, \text{ i.e. } \frac{\dot{\lambda}}{\lambda} = -\sigma \frac{\dot{c}}{c}, \tag{31}$$

$$\frac{\dot{\lambda}}{\lambda} = \rho - r, \tag{32}$$

$$\lim_{t \to \infty} \lambda_t e^{-(\rho - n)t} a_t = 0. \tag{33}$$

Combining (31) with (32), we obtain the standard Keynes-Ramsey rule

$$\frac{\dot{c}}{c} = \frac{r - \rho}{\sigma}. \tag{34}$$

Using $\dot{N}/N = n$ together with definitions $\tilde{c} = c/N^{\frac{1-\theta}{1-\phi}}$ and $g = \frac{(1-\theta)n}{1-\phi}$ confirms (27). In a similar fashion, (24) can be derived from (4).

Moreover, use $l^Y = 1 - l^I$, $\dot{N}/N = n$, $\tilde{k} = k/N^{\frac{1-\theta}{1-\phi}}$ and $g = \frac{(1-\theta)n}{1-\phi}$ in (6) to confirm (26).

According to (3), the inverse demand function for intermediate good i reads $p_i = \alpha(L^Y/x_i)^{1-\alpha}$. Combining with (8) yields

$$x_i = x = \left(\frac{\alpha}{\kappa(1 - s_K)(r + \delta)} \right)^{\frac{1}{1-\alpha}} L^Y. \tag{35}$$

Using the facts that $y/k = (Al^Y/k)^{1-\alpha}$ and $k = Ax/N$, we find

$$\frac{y}{k} = \frac{\kappa(1 - s_K)(r + \delta)}{\alpha}. \tag{36}$$

Substituting (5) into (36) and using that $A/k = \tilde{A}/\tilde{k}$ then confirms (28).

Next, substitute (8) and (35) into profit function $\pi_i = [p_i - (1 - s_K)(r + \delta)]\, x_i$ to obtain the following expression for the profit of each intermediate goods producer i:

$$\pi_i = \pi = (\kappa - 1) \left(\frac{\alpha}{\kappa} \right)^{\frac{1}{1-\alpha}} [(1 - s_K)(r + \delta)]^{-\frac{\alpha}{1-\alpha}} L^Y. \tag{37}$$

Now recall definition $p^A = P^A/N$ as well as $L^Y/N = l^Y = 1 - l^I$ to rewrite capital market equilibrium condition $\dot{P}^A/P^A + \pi/P^A = r$ such as to confirm (25).

Since final goods producers take the wage rate as given, we have $w = (1-\alpha)Y/L^Y$. Thus,

$$w = (1-\alpha)A^{1-\alpha}\left(\frac{k}{l^Y}\right)^{\alpha}, \tag{38}$$

according to (5) and the fact that $Y/L^Y = y/l^Y$.

Due to free entry in the R&D sector, in equilibrium, $\Pi = P^A\dot{A} - (1-s_A)\,wL^I = 0$ holds. Using $\tilde{\nu} = \nu\left(L^I\right)^{-\theta}$ in (4), we have

$$\dot{A} = \nu A^{\phi}\left(L^I\right)^{1-\theta}; \tag{39}$$

thus, $p^A\nu A^{\phi}N^{1-\theta}(l^I)^{-\theta} = (1-s_A)w$. Inserting (38) implies

$$(1-\alpha)A^{1-\alpha}\left(\frac{k}{l^Y}\right)^{\alpha} = \frac{p^A\nu A^{\phi}N^{1-\theta}(l^I)^{-\theta}}{1-s_A}. \tag{40}$$

Using the definitions of $\tilde{k}$ and $\tilde{A}$ (thus, $A^{\phi-1}N^{1-\theta} = \tilde{A}^{\phi-1}$), we then obtain

$$(1-\alpha)\left(\frac{\tilde{k}}{\tilde{A}l^Y}\right)^{\alpha} = \frac{p^A\nu\tilde{A}^{\phi-1}(l^I)^{-\theta}}{1-s_A}. \tag{41}$$

Substituting $l^Y = 1 - l^I$ into (41) confirms (29).

Finally, using $\dot{\lambda}/\lambda = -\sigma g$ from (31) and $\dot{c}/c = g$, we find that if a grows with rate g in the long run, the transversality condition (33) holds under assumption (A1). In fact, total asset holding per capita is $a = k + p^A A$. Thus, in steady state, a grows at the same rate g as k and A. *Q.E.D.*

Proof of part (a) of Proposition 1. The social planner maximizes the household's utility stream subject to (6) and (39). The corresponding current-value Hamiltonian is given by

$$\mathcal{H} = \frac{c^{1-\sigma}-1}{1-\sigma} + \lambda_k(\overbrace{\underbrace{k^\alpha (Al^Y)^{1-\alpha}}_{=y} - (\delta+n)k - c}^{\dot{k}}) + \lambda_A \overbrace{\nu A^\phi N^{1-\theta} (\underbrace{1-l^Y}_{=l^I})^{1-\theta}}^{=\dot{A}}, \quad (42)$$

where λ_k and λ_A are the respective co-state variables. Necessary optimality conditions are $\partial\mathcal{H}/\partial c = \partial\mathcal{H}/\partial l^Y = 0$ (control variables), $\dot{\lambda}_z = (\rho - n)\lambda_z - \partial\mathcal{H}/\partial z$ for $z \in \{k, A\}$ (state variables), and the corresponding transversality conditions. Thus,

$$\lambda_k = c^{-\sigma}, \text{ i.e. } \frac{\dot{\lambda}_k}{\lambda_k} = -\sigma\frac{\dot{c}}{c}, \quad (43)$$

$$(1-\alpha)A^{1-\alpha}\left(\frac{k}{l^Y}\right)^\alpha = \frac{\lambda_A}{\lambda_k}(1-\theta)\underbrace{\nu A^\phi N^{1-\theta}(l^I)^{-\theta}}_{=\dot{A}/l^I}, \quad (44)$$

$$\frac{\dot{\lambda}_k}{\lambda_k} = \rho - \alpha\left(\frac{Al^Y}{k}\right)^{1-\alpha} + \delta, \quad (45)$$

$$\frac{\dot{\lambda}_A}{\lambda_A} = \rho - n - \frac{\lambda_k}{\lambda_A}(1-\alpha)\underbrace{\left(\frac{k}{A}\right)^\alpha (l^Y)^{1-\alpha}}_{=y/A} - \phi\frac{\dot{A}}{A} \quad (46)$$

$$\lim_{t\to\infty} \lambda_{z,t} e^{-(\rho-n)t} z_t = 0, \;\; z \in \{k, A\}. \quad (47)$$

($\lambda_{z,t}$ denotes the co-state variable associated with state variable z at time t.)

To find the optimal capital cost subsidy, first note that from (31) and (43) that we must have $\lambda = \lambda_k$ in social optimum; thus, according to (32) and (45),

$$r = \alpha\left(\frac{Al^Y}{k}\right)^{1-\alpha} - \delta. \quad (48)$$

Comparing (48) with (28), by using $l^Y = 1 - l^I$ and the definitions of $\tilde{c}$, $\tilde{k}$, $\tilde{A}$, we find that $\kappa(1 - s_K) = 1$ must hold in social optimum at all times, which is equivalent to (13).

Next, note from (40) and (44) that a R&D subsidy which implements the social optimum must fulfill

$$p^A = (1-\theta)(1-s_A)\frac{\lambda_A}{\lambda_k}. \tag{49}$$

Moreover, substituting optimality conditions $1 - s_K = 1/\kappa$ and (49) into (25), we find

$$\frac{\dot{p}^A}{p^A} = r - n - \frac{(1-1/\kappa)\alpha(1-l^I)}{(Al^Y/k)^\alpha (1-s_A)(1-\theta)}\frac{\lambda_k}{\lambda_A}. \tag{50}$$

Rewriting (44) and using $\dot{A}/A = g$ leads to

$$\frac{\lambda_k}{\lambda_A} = \frac{(1-\theta)\left(Al^Y/k\right)^\alpha g}{(1-\alpha)l^I} \tag{51}$$

Substituting (51) into (50) leads to

$$\frac{\dot{p}^A}{p^A} = r - n - \frac{(1-1/\kappa)g}{(1-s_A)(1/\alpha-1)}\frac{1-l^I}{l^I}. \tag{52}$$

Moreover, combining (45) and (46) by subtracting both sides of the equations from each other, using $\dot{A}/A = g$ and substituting (51), we have

$$\frac{\dot{\lambda}_A}{\lambda_A} - \frac{\dot{\lambda}_k}{\lambda_k} = \alpha\left(\frac{Al^Y}{k}\right)^{1-\alpha} - \delta - n - (1-\theta)\frac{l^Y}{l^I}g - \phi g. \tag{53}$$

From (43) and $\dot{c}/c = g$, we find $\dot{\lambda}_k/\lambda_k = -\sigma g$; combining with (45) implies

$$\alpha\left(\frac{Al^Y}{k}\right)^{1-\alpha} - \delta = \sigma g + \rho. \tag{54}$$

From (51) and $\dot{A}/A = g$, together with the property that Al^Y/k are l^A are constant in the long run, we find $\dot{\lambda}_k/\lambda_k = \dot{\lambda}_A/\lambda_A$. Using $\dot{\lambda}_k/\lambda_k = \dot{\lambda}_A/\lambda_A$, $\dot{A}/A = g$, $(1-\phi)g = (1-\theta)\,n$, (54) and $l^Y = 1 - l^I$ in (53) implies

$$\frac{1-l^I}{l^I} = \frac{(\sigma-1)g + \rho - \theta n}{(1-\theta)g}. \tag{55}$$

Substituting (55) into (52) and setting $\dot{p}^A = 0$ yields

$$1 - s_A = \frac{1 - 1/\kappa}{1/\alpha - 1} \frac{(\sigma - 1)g + \rho - \theta n}{(1 - \theta)(r - n)}. \tag{56}$$

Combining (48) and (54) implies that, in the long run, $r = \sigma g + \rho$. Using this in (56) confirms (14).

Finally, using that $\dot{\lambda}_k/\lambda_k = \dot{\lambda}_A/\lambda_A = -\sigma g$ for $t \to \infty$ and $\dot{k}/k = \dot{A}/A = g$, transversality condition (47) is fulfilled under assumption (A1) for both state variables, k and A.

B. The Romer (1990) model

Proposition B.1. *Suppose that $\phi = 1$ and $n = \theta = 0$ hold. Then the evolution of the market economy is governed by the following dynamic system (in addition to appropriate boundary conditions):*

$$\dot{A} = \nu N A l^I, \tag{57}$$

$$\dot{p}^A = r p^A - \frac{(\kappa - 1)(\alpha/\kappa)^{\frac{1}{1-\alpha}}(1 - l^I)}{[(1 - s_K)(r + \delta)]^{\frac{\alpha}{1-\alpha}}}, \tag{58}$$

$$\dot{k} = k^\alpha \left(A(1 - l^I)\right)^{1-\alpha} - c - \delta k, \tag{59}$$

$$\frac{\dot{c}}{c} = \frac{r - \rho}{\sigma}, \tag{60}$$

$$r + \delta = \frac{\alpha}{\kappa(1 - s_K)} \left(\frac{A(1 - l^I)}{k}\right)^{1-\alpha}, \tag{61}$$

$$p^A = \frac{(1 - s_A)(1 - \alpha)}{\nu N} \left(\frac{k}{A(1 - l^I)}\right)^\alpha. \tag{62}$$

Proof of Proposition B.1. First, note that (31)-(40) still hold when setting $\phi = 1$ and $n = \theta = 0$. Thus, (57) follows from (11) by setting $\phi = 1$ and $\theta = 0$, (58) follows from (25) by setting $n = 0$, (59) follows from (6) by setting $n = 0$ and using $l^Y = 1 - l^I$, (60) restates (34), (61) follows from substituting (5) into (36), and (62) follows (40) by setting $\phi = 1$ and $\theta = 0$. *Q.E.D.*

Proof of part (b) of Proposition 1. First, note that (43)-(48) still hold when setting $\phi = 1$ and $n = \theta = 0$. (61) and (48) coincide when we set $s_K = s_K^* = 1 - 1/\kappa$.

To prove the result on the optimal long run R&D subsidy, note that $\phi = 1$ and $\theta = 0$ implies that (44) be written as

$$\frac{\lambda_k}{\lambda_A} = \frac{\nu N}{1-\alpha}\left(\frac{Al^Y}{k}\right)^{\alpha}. \tag{63}$$

Substituting (57) and (63) into (46) and using $n = 0$, $\phi = 1$ and $l^Y + l^I = 1$, we obtain

$$\frac{\dot{\lambda}_A}{\lambda_A} = \rho - \nu N. \tag{64}$$

In a steady state where A and k grow at the same rate and $\dot{l}^Y = 0$, it holds that λ_k and λ_A grow at the same rate, according to (63). Thus, we can set the right-hand side of (45) equal to the right-hand side of (64), which leads to $\alpha\left(Al^Y/k\right)^{1-\alpha} - \delta = \nu N$. Thus, according to (48), the optimal long run interest rate in the economy should be $r = \nu N =: r^{**}$. Using that $\frac{\dot{c}}{c} = \frac{\dot{A}}{A}$ in decentralized long run equilibrum, (57) and (60) imply

$$r = \sigma\nu N l^I + \rho := \tilde{r}. \tag{65}$$

Setting $\tilde{r} = r^{**}$ implies that the socially optimal long run fraction of labor allocated to R&D reads as

$$l^I = \frac{1}{\sigma}\left(1 - \frac{\rho}{\nu N}\right) =: l^{I**}. \tag{66}$$

We seek for the R&D subsidy rate which implements a long run fraction of R&D labor equal to l^{I**}. Substituting (61) and (65) into (58) and setting $\dot{p}^A = 0$ leads to

$$p^A = \frac{(1-1/\kappa)\alpha(1-l^I)}{\sigma\nu N l^I + \rho}\left(\frac{k}{A(1-l^I)}\right)^{\alpha}. \tag{67}$$

Combining (67) with (62) we obtain

$$s_A = 1 - \frac{1-1/\kappa}{1/\alpha - 1}\frac{1-l^I}{\sigma l^I + \frac{\rho}{\nu N}}. \tag{68}$$

Substituting (66) into (68) confirms (15). *Q.E.D.*

Calibration of νN. Substituting $l^I = \frac{g}{\nu N}$ into (68) leads to

$$\nu N = (\sigma g + \rho)(1 - s_A)\frac{1/\alpha - 1}{1 - 1/\kappa} + g. \tag{69}$$

Thus, with $s_A = 0.066$, $\alpha = 0.33$, $\sigma = 2.5$, $\kappa = 1.4$, $g = 0.02$, $\rho = 0.02$, we calibrate $\nu N = 0.48$.

Steady state growth rate. By solving (68) for l^I one obtains

$$l^I = \frac{(1 - 1/\kappa)\,\nu N - (1 - s_A)\,(1/\alpha - 1)\,\rho}{1 - 1/\kappa + (1 - s_A)\,(1/\alpha - 1)\,\sigma\nu N} \tag{70}$$

Combining $\dot{A}/A = \nu N l^I$ from (57) with (70) yields (10).